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The Advantages of Bringing Infinity to a Finite Place: Penrose Diagrams as Objects of Intuition

ABSTRACT

The history of Penrose diagrams in the physics of General Relativity (GR) is presented. It is argued that the diagrams did conceptual work for physicists, providing a literal *place* for abstract, formal objects. Penrose diagrams were associated with the mathematics of conformal transformations applied to GR. Together the diagrams and formalism reconfigured the basic concepts of the field—notions of space, time, cosmology, and energy. Nor were the meanings of the diagrams themselves stable over time. Their physical and conceptual evolution is traced. This history also demonstrates the tight integration of the contexts of research and pedagogy in the period investigated (1962–66). Diagrams circulated rapidly between research talks and publications and the pedagogical context of summer school lectures for advanced graduate students. Further reception and circulation of the diagrams is briefly examined.

KEY WORDS: General relativity, Penrose diagrams, space-time diagrams, diagrams, research, pedagogy

INTRODUCTION

What does it mean to think with a picture? Why do physicists draw diagrams alongside the words and mathematical formalisms that mark their work? These are the questions that animate this discussion. And they are large questions. In order to be concrete, this paper takes up the example of Penrose diagrams—a type of diagram used in the field of physics called General Relativity (GR). They were developed from 1962 to 1966 by Roger Penrose (born 1931 in England) soon after he completed his PhD at Cambridge University in 1957.

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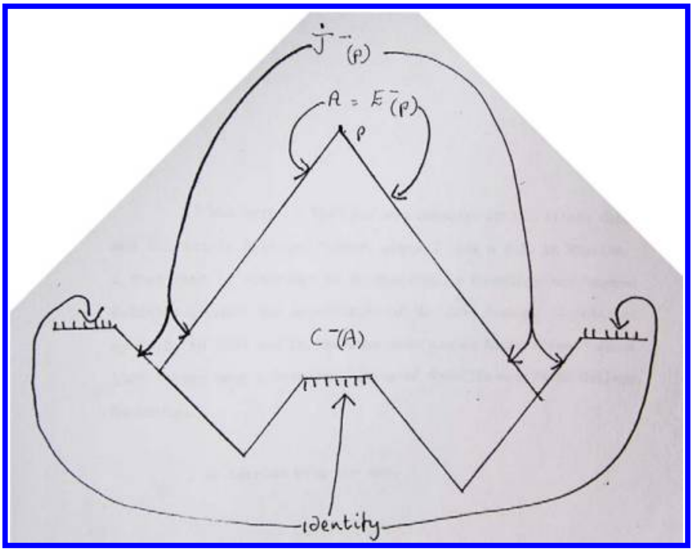

FIG 1. Penrose diagram (1968). *Source*: Hawking and Penrose, "Gravitational Collapse and Cosmology" (ref. 2). Courtesy Niels Bohr Archives, American Institute of Physics, reproduced with permission.

Through a careful exposition of Penrose's introduction of and changing work with the diagrams, it is argued that they are best understood as facilitating physicists' *conceptual* work. Tracing the diagrams' early uses also reveals an astonishing closeness between contexts of research and pedagogical work in the field of GR during this time. In a matter of months the diagrams moved from research presented in a conference paper and a top-tier journal to the pedagogical context of a "summer school" for young physicists. By 1968, soon after the main analysis of this paper ends, Penrose diagrams were used in work that would eventually produce among the most famous results in modern physics: Stephen Hawking and Penrose's discussion of the inevitability of the creation of singularities by gravitational collapse.[1] Fig. 1 shows a diagram accompanying Hawking and Penrose's submission to a paper prize in 1968, probably in Hawking's hand.[2] Though a thorough exposition of these diagrams is beyond

1. John Earman, "The Penrose-Hawking Singularity Theorems: History and Implications," in *The Expanding Worlds of General Relativity*, ed. Hubert Goenner et al., vol. 7, Einstein Studies (Boston: Birkauser, 1999), 235–67.

2. Stephen Hawking and Roger Penrose, "On Gravitational Collapse and Cosmology," Gravity Research Foundation, American Institute of Physics, Wellesley Hills, MA, Box 16, Folder 1968 Competition.

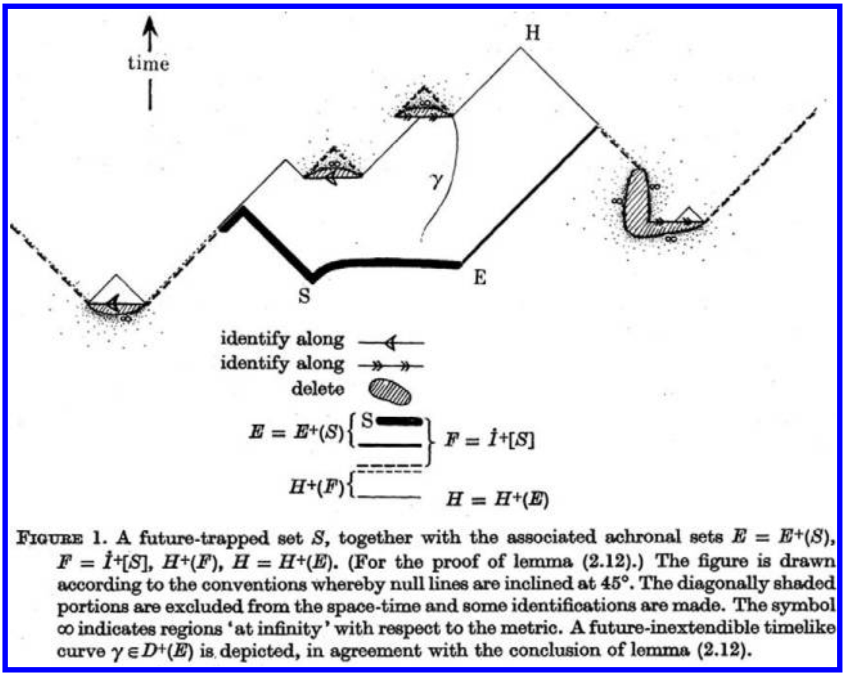

FIG 2. Penrose diagram (1970) from Hawking and Penrose, "Singularities of Gravitational Collapse" (ref. 3), 537. Copyright Royal Society of London, reproduced with permission.

the scope of this paper, a related diagram in the published version of their work is shown in Fig. 2.[3] By 1973 the diagrams were included in monographs such as Hawking and Ellis's *The Large-Scale Structure of Space-Time* and Misner, Thorne, and Wheeler's *Gravitation*.[4] The adoption of the diagrams was not universal, however. Steven Weinberg's 1972 *Gravitation and Cosmology* contains almost no diagrams of any sort, and no Penrose diagrams. But Weinberg's choice to elide the diagrams and their accompanying mathematics circumscribed the topics he was able to cover (though he also attributed this to a lack of space). "The nongeometrical approach taken in [Weinberg's] book has, to

3. S. W. Hawking and R. Penrose, "The Singularities of Gravitational Collapse and Cosmology," *Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences* 314, no. 1519 (1970): 537.

4. S. W. Hawking and G. F. R. Ellis, *The Large-Scale Structure of Space-Time*, Cambridge Monographs on Mathematical Physics (Cambridge: Cambridge University Press, 1973); Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler, *Gravitation* (London: W. H. Freeman, 1973).

some extent, affected the choice of the topics . . . covered. . . . [He] regret[s] the omission here of a detailed discussion of the beautiful theorems of Hawking and Penrose on gravitational collapse."[5]

Turning now to the history of GR, Albert Einstein's death in 1955 marked an inflection point in the historical trajectory of his theory of gravity.[6] Einstein introduced GR—in a modern form—to the *Königlich Preußische Akademie der Wissenschaften* in Berlin in 1915 and 1916 using an unfamiliar and difficult form of mathematics. He argued that gravity was not a conventional force like electromagnetism, but was a manifestation of the curvature of the geometry of space-time.[7] Einstein and GR were rocketed to international fame by Arthur Eddington's 1919 eclipse expeditions, which confirmed Einstein's prediction of the bending of rays of light around massive objects, and by Einstein's 1921 Nobel Prize for Physics (awarded in 1922). However, GR's fortunes rapidly fell. As historian Jean Eisenstaedt has put it, the years 1925–55 were GR's "low water mark."[8] The theory was seen as difficult to understand, difficult to interpret, and difficult to connect to experiment or to the rest of physics. It also lacked an institutional place within physics. Yet, after 1955, the theory slowly regained physicists' interest, the number of papers devoted to the subject grew, and during the 1960s the field experienced a "renewal."[9] Physicist Clifford Will has christened this period the "renaissance" of general relativity.[10]

5. Steven Weinberg, *Gravitation and Cosmology* (New York: Wiley, 1972), viii.

6. David Kaiser, "A ψ is just a ψ? Pedagogy, Practice, and the Reconstitution of General Relativity, 1942–1975," *Studies in History and Philosophy of Science, Part B: Studies in History and Philosophy of Modern Physics* 29, no. 3 (1998): 321–38.

7. For a standard account see Abraham Pais, *Subtle Is the Lord: The Science and Life of Albert Einstein* (Oxford: Oxford University Press, 1982), sec. IV. The papers and an excellent editorial introduction are available in Albert Einstein, *The Collected Papers of Albert Einstein,* vol. 6, *The Berlin Years: Writings, 1914–1917*, ed. A. J. Kox, Martin J. Klein, and Robert Schulmann (Princeton, NJ: Princeton University Press, 1996). More recent scholarship is collected in Michel Janssen et al., eds., *The Genesis of General Relativity*, vol. 250, Boston Studies in the Philosophy of Science (New York: Springer, 2007).

8. Jean Eisenstaedt, "The Low Water Mark of General Relativity, 1925–1955," in *Einstein and the History of General Relativity: Based on the Proceedings of the 1986 Osgood Hill Conference, North Andover, Massachusetts, 8–11 May 1986*, ed. Don Howard and John J. Stachel (Boston: Birkhaüser, 1989), 277–92.

9. Though not as a percentage of the massively growing physics literature. Jean Eisenstaedt, "La relativité générale à l'étiage: 1925–1955," *Archive for History of Exact Sciences* 35, no. 2 (1986): 179, on 183.

10. Clifford Will, "The Renaissance of General Relativity," in *The New Physics*, ed. P. C. W. Davies (Cambridge: Cambridge University Press, 1989), 7–33. The term has also been applied specifically to Dennis Sciama's group at Cambridge University, which included S. Hawking,

What explains this "renaissance"? Will argues that experiment and observation played the central role in the decline and rebirth of GR: "Largely because of [the] paucity of experimental contact" until the late 1950s "the science of general relativity became stagnant"; and in the "renaissance . . . experiment and observation motivated and complemented theoretical advances."[11] However, we have reasons to question the centrality of new empirical evidence for the "renaissance" of GR. Early observational results, while spectacular, were not in contact with theoretical details. As late as 1963 Peter Bergmann reflected that meaningful connection between theory and observation was at least five years away.[12] In 1965, of the nearly 280 members of the International Committee on General Relativity and Gravitation, only about twelve listed experiment as an area of research.[13] We can sense the mood between theory and experiment from a conference report from 1965: "experimentalists do not attach too much importance to 'theories made to corroborate facts'. . . . Yet in such fields as those studies at this Symposium, it is not silly to put it that way rather than the other way round ('facts to corroborate theories'), for the observations made and reported lack considerably in preciseness and thus the only thing one is more or less sure of is the rigor of mathematical deduction."[14] While experiment and observation were important, and inspired workers in GR at one broad level, this distance between theory and experiment motivates a closer look at the features of the development of GR theory in the explanation of GR's "renaissance."

R. Penrose, and B. Carter with varying degrees of formality. George Ellis et al., *The Renaissance of General Relativity and Cosmology: A Survey to Celebrate the 65th Birthday of Dennis Sciama* (Cambridge: Cambridge University Press, 1993).

11. Will does include theoretical advances in his discussion, but maintains the central place of experiment as the motivator for theory throughout the article. Will, "Renaissance of General Relativity" (ref. 10), 7.

12. Peter G. Bergmann, "Summary," chap. 35 in *Quasi-Stellar Sources and Gravitational Collapse, Including the Proceedings of the First Texas Symposium on Relativistic Astrophysics*, ed. Ivor Robinson, Alfred Schild, and Engelbert L Schücking (Chicago: University of Chicago Press, 1965), 431–32.

13. International Society on General Relativity and Gravitation, Records 1961–1982, Niels Bohr Library and Archive, American Institute of Physics, College Park, MD, Box 1, Folders 7–8 and Box 2, Folder 9; Eisenstaedt, "La relativité générale" (ref. 9), 181.

14. A. M[ercier], "Report on the Second Texas Symposium on Relativistic Astrophysics," *Bulletin of General Relativity and Gravitation* no. 8 (1965), p. 8, International Society on General Relativity and Gravitation, Records 1961–1982, Niels Bohr Library and Archive, American Institute of Physics, College Park, MD, Box 1, Folder 8.

It is worth a moment to pause at the invocation of mathematical rigor here. This stands in stark contrast to other uses of diagrammatic reasoning in the physical sciences.[15] For example, as Andrea Woody has detailed, the introduction of diagrammatic techniques in quantum chemistry went hand in hand with approximation techniques.[16] Similarly, debates on the "realism" of Feynman diagrams in particle physics can be construed as debates over their accuracy and rigor.[17] Penrose diagrams played into André Mercier's confidence in the "rigor of mathematical deduction." At the same time they exhibit the diversity of uses to which diagrammatic reasoning has been put in the sciences.

This paper will exhibit the close connection between research and pedagogy in GR. This contributes to a growing literature in the history of science. David Kaiser has argued that specialized scientific training is a universal quality of the modern sciences that must be taken seriously, and that can provide a common touchstone for historians of science.[18] Kaiser and Andrew Warwick have emphasized the under-examined role of pedagogy in the philosophy of Kuhn and Foucault, while Kathryn Olesko has directed attention to Ludwig Fleck.[19] Studying pedagogy is one way to uncover the thinking behind the published record of the history of physics.[20] The way of thinking investigated by this paper surrounds Penrose diagrams, a particular tool that physicists and cosmologists used to understand general relativity and cosmology. These diagrams depicted universes and black holes, objects that were unobservable in important ways and

15. I thank Ursula Klein for making this point.

16. Andrea Woody, "Putting Quantum Mechanics to Work in Chemistry: The Power of Diagrammatic Representation," *Philosophy of Science* 67, suppl. 1 (2000): S612–27; Andrea Woody, "Concept Amalgamation and Representation in Quantum Chemistry," in *Handbook of the Philosophy of Science,* vol. 6, *Philosophy of Chemistry* (Amsterdam: Elsevier, 2010), 1–42.

17. David Kaiser, "Stick-Figure Realism: Conventions, Reification, and the Persistence of Feynman Diagrams, 1948–1964," *Representations* 70 (2000): 49–86.

18. David Kaiser, ed., *Pedagogy and the Practice of Science: Historical and Contemporary Perspectives* (Cambridge, MA: MIT Press, 2005); David Kaiser, "Training and the Generalist's Vision in the History of Science," *Isis* 96, no. 2 (2005): 244–51. Though the death of amateur science may be premature, see Patrick McCray, *Keep Watching the Skies!: The Story of Operation Moonwatch and the Dawn of the Space Age* (Princeton, NJ: Princeton University Press, 2008).

19. Andrew Warwick, *Masters of Theory: Cambridge and the Rise of Mathematical Physics* (Chicago: University of Chicago Press, 2003); Andrew Warwick and David Kaiser, "Conclusion: Kuhn, Foucault, and the Power of Pedagogy," in Kaiser, *Pedagogy* (ref. 18), 393–410; Kathryn Olesko, "Science Pedagogy as a Category of Historical Analysis: Past, Present, and Future," *Science and Education* 15, no. 7 (2006): 863–80.

20. We know that visual thinking can sometimes hide beneath the surface of physicists' published thought. Peter Galison, "The Suppressed Drawing: Paul Dirac's Hidden Geometry," *Representations* 72 (2000): 145–66.

that defied physical intuitions about space and time. "Universe" and "black hole" are doubled concepts—they are thought of as both physical places and as mathematical objects. Unlike Richard Feynman's diagrams that are used in atomic and nuclear physics, Penrose diagrams should not be construed as calculational, mnemonic tools.[21] As will be developed below, Penrose diagrams did more conceptual work; they served to reorient the physicist who had manipulated a formalism, but did not see the relationship between the concepts the formalism represents. They reestablished the formalism's place.

Penrose diagrams were part of the practice and material culture of theoretical physics. Paper and pen, slate and chalkboard. These are hardly as exciting as particle accelerators; however, they have had enormous import. Andrew Warwick has shown how the introduction of (affordable) pens and paper in eighteenth-century Cambridge changed the form and content of mathematical physics.[22] Penrose diagrams were not a new media in which to do physics, but they were an important new *way* to use them. In Warwick's terminology they are a "theoretical technology." With Ursula Klein and David Kaiser I call them "paper tools."[23] Penrose diagrams were introduced to physics in 1962 as part of a new method of applying topology to problems in GR—they were part of a toolkit that included other formal techniques such as conformal transformations and point-set topology. Penrose diagrams cannot be analyzed without the formalisms surrounding them. Because of this, studying the diagrams can tell us about the relationship between traditional formal-symbolic reasoning and visual reasoning. To material culture we can add visual culture as an object of investigation of humanistic studies of science.[24]

THE INTRODUCTION OF PENROSE DIAGRAMS

In another paper I have detailed some of the prehistory of Penrose diagrams, in particular Penrose's engagement with M. C. Escher's art and the psychology of

21. Feynman diagrams were more than calculational tools, of course. David Kaiser, *Drawing Theories Apart: The Dispersion of Feynman Diagrams in Postwar Physics* (Chicago: University of Chicago Press, 2005).

22. Warwick, *Masters of Theory* (ref. 19), chap. 3.

23. Ursula Klein, *Experiments, Models, Paper Tools: Cultures of Organic Chemistry in the Nineteenth Century* (Stanford, CA: Stanford University Press, 2003); Kaiser, *Drawing Theories Apart* (ref. 21).

24. Alex Soojung-Kim Pang, "Visual Representation and Post-Constructivist History of Science," *HSPS* 28, no. 1 (1997): 139–71.

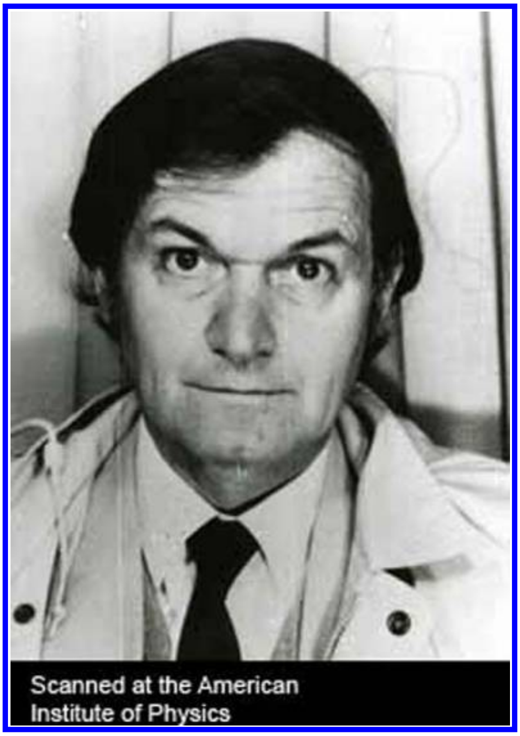

FIG 3. Photograph of Penrose, date unknown. *Source*: Emilio Segre Visual Archives, American Institute of Physics. Courtesy Emilio Segre Visual Archives, American Institute of Physics, reproduced with permission.

the perception of "impossible objects" in the 1950s.[25] Here I restrict myself to the emergence of the actual diagrams and their development from 1962 to 1966. (A photograph of Penrose is shown in Fig. 3). I will introduce another of the visual traditions out of which Penrose diagrams were born—that of drawing space-time diagrams—and exhibit how these new diagrams altered the fundamental concepts upon which regular space-time diagrams were based.

Space-time diagrams have a history in physics dating back to Hermann Minkowski's 1907 elaboration of Einstein's theory of relativity in terms of a unified, four-dimensional picture.[26] Minkowski joined the three regular

25. Aaron Sidney Wright, "The Origins of Penrose Diagrams in Physics, Art, and the Psychology of Perception," *Endeavour* 37 no. 3 (2013): 133–39.

26. See Peter Galison, "Minkowski's Space-Time: From Visual Thinking to the Absolute World," *HSPS* 10 (1979): 85–121; Scott Walter, "Breaking in the 4-Vectors: The Four-

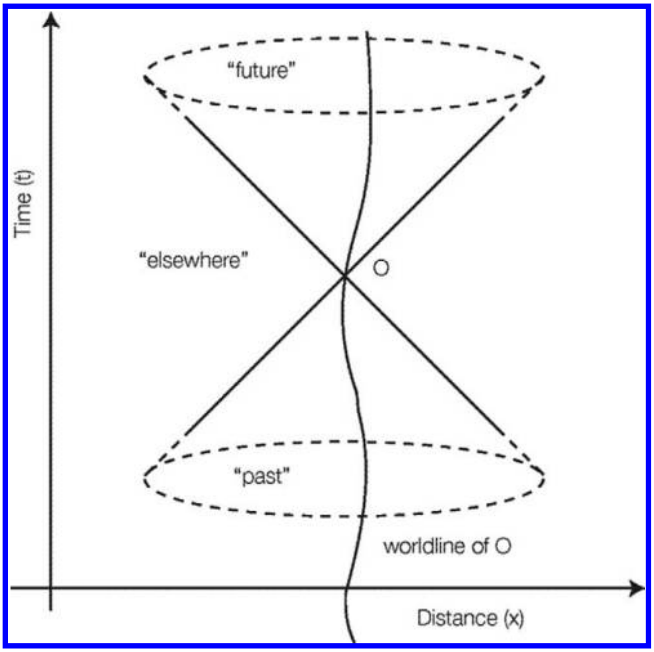

FIG 4. A standard Minkowski diagram with observer *O*, and Penrose's "three regions." *Source*: Author's creation.

spatial dimensions with time. In a space-time diagram time runs along the vertical axis and one spatial dimension runs along the horizontal axis. This leaves two spatial dimensions "suppressed." The paths of "observers" through space-time—whether they be people or galaxies or experimental apparatuses—trace out "world-lines" on the diagram. There is a powerful convention in space-time diagrams that the time axis is understood to be multiplied by c, the speed of light, resulting in a diagram with units of spatial-extent on all axes. This means that the postulate of relativity that light travels at c is rendered graphically by always drawing rays of light at 45°. Then the requirement that nothing travels faster than light becomes the rule that observers' world-lines cannot have a slope of less than 45°, that is, the possible paths of an observer are confined to a "light-cone" with borders at 45°. A simple space-time diagram with an observer, *O*, a world-line, and light-cone is shown in Fig. 4. While we

Dimensional Movement in Gravitation, 1905–1910," in Janssen et al., *Genesis of General Relativity* (ref. 7), 1118–78.

might think that time is divided into three—past, present, and future—in relativity, not only are "past" and "future" relative to each observer, but the unified space-time view reveals a new region that Penrose called "elsewhere" (see below).[27] In Penrose's modern understanding "Minkowski space" was a "manifold" of points with a "metric" that determined how distances are measured.

The Context of Research

Though Fig. 4 shows a type of diagram in the genealogy of Penrose diagrams, his newer diagrams looked different and altered the meanings of the lines on the page. Penrose diagrams and their associated mathematics provided mathematical "rigor" (see below) and conceptual clarity to certain aspects of GR. Moreover, they gave the spatial concepts at play in GR a place. In 1962, while a research associate at King's College, London, Penrose delivered a talk called "The Light Cone at Infinity" at an international conference on relativistic theories in Warsaw, organized by Leopold Infeld. The conference had 114 participants, from 22 countries.[28] This was the first elucidation of the diagrams, in the context of research. To understand the importance of infinity to physics, it is necessary to understand the role of "boundary conditions" applied to physicists' equations. When describing the electromagnetic field surrounding a charge, for example, one can calculate what the magnitude of the field is at any distance from the charge, from a centimeter to a kilometer and beyond. However, the laws of electrostatics state that the field falls off as the inverse of the square of the distance from the charge ($\propto 1/r^2$), so that as distance increases, the strength of the field approaches zero. But if one wants to know the total energy of the field in a mathematically tractable form, one assumes that the field is actually zero "at infinity"; this even though "at infinity" makes little physical sense. Penrose began his paper: "Questions concerning radiation . . . involve statements about events in the 'neighbourhood of infinity'. It would appear . . . that some deeper understanding of the mathematical nature of this 'infinity' might be of great conceptual value to physics." Accordingly, he introduced the idea of applying a "conformal transformation" to the space-time.

27. This terminology did not begin with Penrose; see preceding note.

28. Austria, Belgium, Bulgaria, Canada, Czechoslovakia, East Germany, France, Great Britain, Hungary, Iceland, Ireland, Israel, Italy, Poland, Romania, Sweden, Switzerland, Tasmania, Tunis, United States, USSR, West Germany. Leopold Infeld, ed., *Conférence Internationale sur les Théories Relativistes de la Gravitation: Sous la direction de L. Infeld* (Paris: Gauthier-Villars, 1964), viii–x

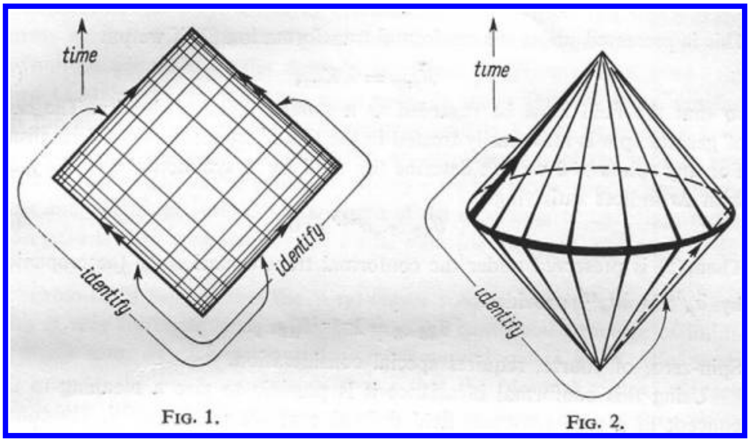

FIG 5. Penrose's illustrations (1964) of conformally transformed Minkowski manifolds, from Penrose, "Light Cone at Infinity" (ref. 29), 371. The publisher of the journal seems to no longer exist. I have made a good-faith effort to procure permissions to print this figure but to no avail.

A conformal transformation is one that preserves the angles between objects but distorts their size; the Mercator projection of the globe of the Earth onto a flat page is a conformal transformation. For Penrose "[t]he idea [was] that if space-time is considered from the point of view of its conformal structure only, points at infinity can be treated on the same basis as finite points."[29]

Penrose discussed some formal aspects of conformal transformations and then asked his audience to "[i]magine the whole of two-dimensional Minkowski space-time to be mapped continuously onto the interior of a square," directing them to Fig. 5 (left). The convention that light moves at 45° dictated that Penrose's square be on point. The hatching within the square indicated that as one moved out from the center of the square, the distance scale shrinks so that "infinity is represented by the sides of the square." The instructions to "identify" different edges of the diagram invite the reader to mentally manipulate the figure and attach the edges such that the "resultant compact manifold is topologically a torus" (like a doughnut). The conformal transformation had allowed infinity to be placed on the page at the price of this change in topology from a "flat" Minkowski space-time to a torus. But changes in topology reconfigured the basic concepts of Minkowski space. After the transformation

29. Roger Penrose, "The Light Cone at Infinity," in Infeld, ed., *Conférence Internationale* (ref. 28), 369.

"the three regions 'past', 'future', 'elsewhere' into which a [light-]cone divides normal Minkowski space [see Fig. 4] are *connected* to each other." In the conformally transformed space, there was no certain distinction between past and future, or between regions an observer could travel to, and regions that can only be reached by traveling faster than light. "There [was], thus, no invariant distinction between a space-like or a time-like separation for two general points in" the conformally transformed manifold.[30] These were the types of major conceptual shifts Penrose's treatment engendered—infinity was made tractable and place-able at the cost of reconfiguring the basic concepts of relativistic space and time.

Soon after the 1962 Warsaw conference—still in a research context—Penrose submitted a paper to *Physical Review Letters* (*PRL*) refining the conceptual shifts of his conformal techniques and their diagrammatic accompaniments.[31] *PRL* was the flagship journal of the American Physical Society—as a "letters" journal, its brief reports were supposed to make the most important new research in physics accessible to the widest audience of English-reading physicists. The meanings of the diagrams were not perfectly consistent even within the contexts of research presentations and publications in 1962. In *PRL*, Penrose no longer insisted that his mathematical treatment demanded "identifying" the pasts and futures of conformally transformed Minkowski space-time. Rather than feeling constrained by the mathematical rules he adhered to at Warsaw, Penrose noted that one can simply "remove" (i.e., ignore) a light-cone of points that severs the earlier identifications. For the physicist-audience of *PRL*, there could be distinctions between "past," "future," and "elsewhere." However, the concepts of past and future were not returned to their previous state. Rather, the concept of infinitely far away was broken up into five pieces according to whether "far away" was in the future, past, or "elsewhere" or was accessible only to particles traveling at the speed of light. Penrose relayed the "basic idea" of his work as arising from the puzzle that (as above) he wanted to be able to deal with quantities at infinity even though "there is no such thing as a point at infinity. . . . But if we think only in terms of conformal structure of space-time (only ratios of neighboring infinitesimal distances are to have significance), then infinity can be treated as though it were simply an ordinary three-dimensional boundary $\mathscr{I}$ [pronounced 'scri'] to a finite four-dimensional conformal

30. Ibid., 369—370, 371, 371, and 371, resp.

31. Roger Penrose, "Asymptotic Properties of Fields and Space-Times," *Physical Review Letters* 10, no. 2 (1963): 66–68.

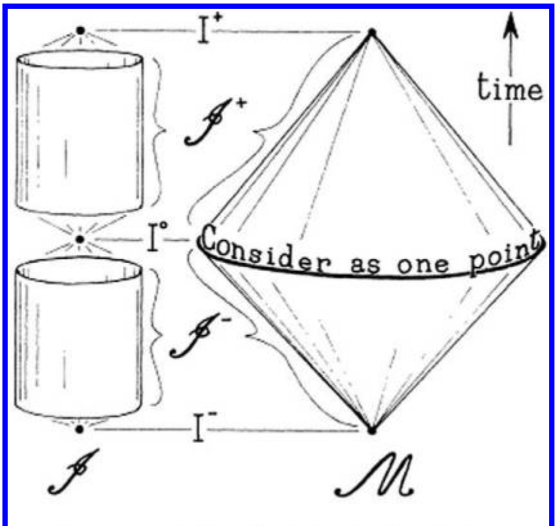

FIG 6. Penrose's caption: "Conformal Structure of Infinity" for Minkowski space-time, from Penrose, "Asymptotic Properties" (ref. 31), 67. Copyright © 1963 American Physical Society, reproduced with permission; see http://prl.aps.org/abstract/PRL/v10/i2/p66_1.

region."[32] Using the example of Minkowski space, $\mathscr{M}$, Penrose wrote that this boundary "$\mathscr{I}$ can be separated into five distinguishable disjoint parts" depicted in Fig. 6: three infinite distances for objects not traveling at the speed of light, future infinity I^+, the past infinity I^-, and the "spatial" infinity corresponding to the "elsewhere" direction in Fig. 4, I^0; and two infinite distances $\mathscr{I}^+$ and $\mathscr{I}^-$ representing the future and past infinities for light, respectively.

These figures were the material basis for the physical concepts that Penrose analyzed—they were material on the page and also used the conventions of realistic drawing to more strongly evoke the sense of engaging with a physical object. These diagrams would have been prepared by Penrose himself, or professionally drafted by a draftsperson. Figure 6 has much in common with Fig. 5 (right). The sense that this was akin to a graph is given by the labeling of the time axis. The explicit instructions to "identify" pieces of the diagram are gone—in fact the insistence that this identification is required by the mathematics has

32. Ibid., 66.

dropped out of the presentation. Both Fig. 5 (right) and Fig. 6 were shown in perspective, but the two conic shapes were rendered differently. The earlier figure presented a mathematical form that had "generators" marked—not shading—and was solid with unseen surfaces marked by the dashed lines. This figure was drawn in perspective, but not realistically; it was more of a blueprint than a rendition of some realized form. The later diagram presented in *PRL* was drawn in perspective and shaded realistically, as if the reader were confronted with a spinning top. Rather than mathematical "generators" the cones were drawn opaque and had shading that became denser at the edges, placing an inferred light-source roughly at the location of the reader. The same was true of the cylinders on the left side. These details are seemingly irrelevant to the scientific content of the image—but this irrelevance precisely marks their role as a sort of visual "reality effect," to borrow from Barthes's literary theory. The text is more closely integrated into Fig. 6, with the instruction to "Consider as one point" wrapped around the equator of the cones, labeled I^0. This instruction was not so bizarre as it may seem. Recall that the Mercator projection is a conformal transformation; we know that the south pole of the Earth is a point on the globe, but this point is stretched into a line stretching across the bottom of an atlas. I^0 was drawn on the left side of the figure as a point, as were I^+ and I^-. On the left then, the figure showed the pieces of I, three points and two cylinders, and on the right the figure showed the "manifold" of Minkowski space, $\mathscr{M}$, with its boundaries connected together. Together they gave a definite place—an objecthood, a materiality—to the mathematical entities that were the formal objects of GR. But despite the formality of mathematical physics, it was connected to physical ideas. Minkowski space is of interest because of its role in understanding physical processes such as radiation. Here, a physicist passed the concept of the space-time of physical processes to a mathematical object, $\mathscr{M}$, which was then partially rematerialized in the form of a realistic diagram. This representation partially constituted the rather abstract mathematical object $\mathscr{M}$, and opened it up to physical intuition.

Continuing the discussion of the diagrams in a research context, during the spring of 1963 Penrose presented at a conference on "the nature of time" organized by Herman Bondi and Thomas Gold at Cornell University. It was held May 30–June 1 and was funded by the U.S. Air Force Office of Scientific Research. Attendance was limited to twenty-two people to facilitate discussion among the participants. Penrose's paper, "Cosmological Boundary Conditions for Zero Rest-Mass Fields," for the first time applied his conformal techniques to cosmology, and introduced a new style of diagram to consider what conditions must hold for there to be "particle horizons" or "event horizons."[33] Here again Penrose put GR in a particle physics context. This horizon terminology was introduced by Wolfgang Rindler in 1956 writing in the *Monthly Notices of the Royal Astronomical Society*. He defined "horizon" as "*a frontier between things observable and things unobservable*. (The vague term *things* is here used deliberately.)" He then divided horizons into two types: "An event-horizon, for a given fundamental observer A, is a (hyper-) surface in space-time which divides all events into two non-empty classes: those that have been, are, or will be observable by A, and those that are forever outside A's possible powers of observation." In contrast: "[a] particle-horizon, for any given fundamental observer A and cosmic instant t_o is a surface in the instantaneous 3-space $t = t_o$, which divides all fundamental particles into two non-empty classes: those that have already been observable by A at time t_o and those that have not."[34]

There is not space here to explicate Rindler 's mathematical understanding of these horizons, as distinct from Penrose's more geometrical picture. However, the concepts "fundamental observer" and "fundamental particle" were in tension between their two approaches, and within Penrose's itself. For Rindler, in the context of Robertson-Walker expanding-universe cosmological models, by "particle" he meant "fundamental particles, i.e. the representations of the nebulae in the world-model." To a cosmologist, "fundamental particles" were enormous nebulae that do not accelerate under their own power (and so move on geodesics). "Fundamental observers" were observers (often attributed powers of observation: having seen something, having cognizance, etc.) that remain attached to their nebulae.[35] As will be shown below, Penrose worked within these conventions in the diagrams in the text, but the framing at the beginning of the article contains a very different particle concept, from particle physics. In the second paragraph of the article he wrote that the "relevant" zero rest-mass fields include "neutrino, spin 1/2 . . . photon, spin 1" and "graviton, spin 2," the field and spin language of particle physics.[36] Until the 1990s, all these fields were thought to be massless; that is, they travel at the speed of light. The conflict between this particle-physics definition and Rindler's cosmological-

33. Roger Penrose, "Cosmological Boundary Conditions for Zero Rest-Mass Fields," in *The Nature of Time*, ed. Thomas Gold and Hermann Bondi (Ithaca, NY: Cornell University Press, 1967), 42–54.

34. W. Rindler, "Visual Horizons in World Models," *Monthly Notices of the Royal Astronomical Society* 116 (1956): 663.

35. Ibid., 663ff, and 672, resp.

36. Penrose, "Cosmological Boundary Conditions" (ref. 33), 42.

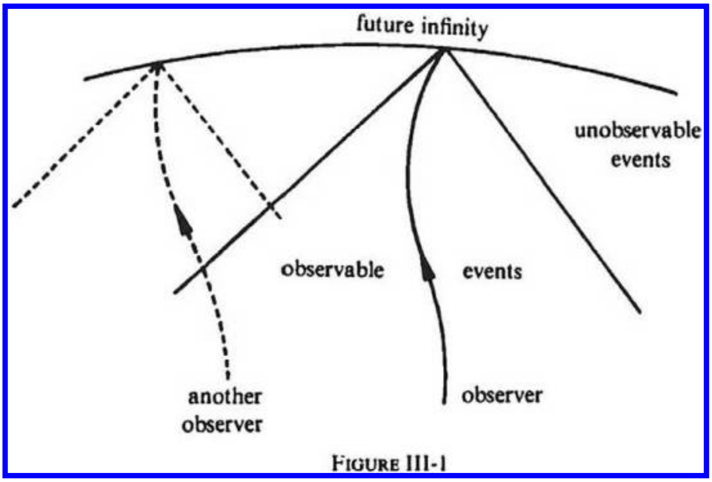

FIG 7. Penrose's (1963 [1967]) diagram of a cosmological event horizon, "future infinity is spacelike." *Source*: Penrose, "Cosmological Boundary Conditions" (ref. 33), 45. Copyright © 1967 by Cornell University. Used by permission of the publisher, Cornell University Press. I have made a good-faith effort to procure permission for e-print to no avail.

particle concept is illustrated by the fact that Penrose depicts his observers and particles as traveling on wavy, vertical lines, breaking the convention that objects at the speed of light travel at 45°.

Penrose sought to clarify the meaning of "advanced" and "retarded" radiative solutions of these particle fields in curved space-time using his conformal techniques.[37] He "stud[ied] fields in [a conformally] transformed space-time instead of in the original physical one." He then used "*local* arguments [to] examine the behaviour of the field at physical infinity." He considered two cosmological models and their manifold of events: the steady-state model of Gold, Bondi, and Hoyle; and the Einstein-de Sitter model.[38] For the steady-state model future infinity was spacelike (horizontal), giving rise to an event horizon (Fig. 7). This diagram was less structured than the earlier, bounded, conformal diagrams. For this audience, Penrose did not introduce his specialized notation, and instead used textual labels ("future infinity" not I° or I^{+}). Consonant with Rindler's definition, the event horizon labeled "future

37. The terminology is standard.

38. See Helge Kragh, *Cosmology and Controversy: The Historical Development of Two Theories of the Universe* (Princeton, NJ: Princeton University Press, 1996).

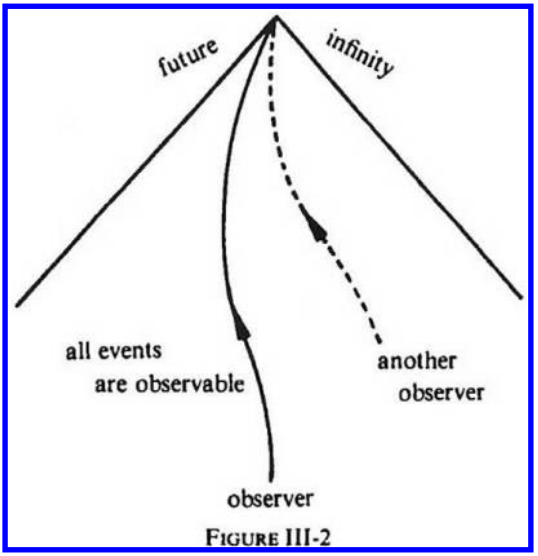

FIG 8. Penrose's (1963 [1967]) diagram of a cosmology without an event horizon, "future infinity is null." *Source*: Penrose, "Cosmological Boundary Conditions" (ref. 33), 45. Copyright © 1967 by Cornell University. Used by permission of the publisher, Cornell University Press. I have made a good-faith effort to procure permission for e-print to no avail.

infinity" divides the manifold of events into "observable events" that will be in the observer's past light (or null) cone at infinity and "unobservable events" that will not. The second observer illustrates Penrose's remark that "the past null cones of spatially separated events near enough to [spacelike future infinity] will not intersect (that is, such events will be 'causally independent' of one another)."[39] Once the two observers are far enough in the future to be above the intersection of their null cones, no event in one cone could affect one in the other (a signal would have to travel horizontally, i.e., faster than light). In contrast, in the Einstein-de Sitter model, future infinity is a null cone; there was no event horizon and all events were observable (Fig. 8).

The situation with the two cosmologies and particle horizons is reversed—it also exhibits further blurring of the cosmological particle with the elementary particle. The Einstein–de Sitter cosmology has a spacelike past infinity, as in

39. Penrose, "Cosmological Boundary Conditions" (ref. 33), 44.

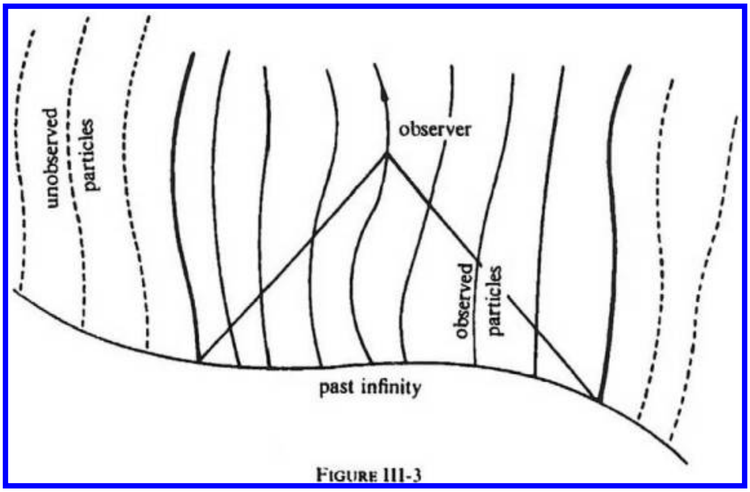

FIG 9. Penrose's (1963 [1967]) diagram of a cosmological particle horizon, "past infinity is spacelike." *Source*: Penrose, "Cosmological Boundary Conditions" (ref. 33), 46. Copyright © 1967 by Cornell University. Used by permission of the publisher, Cornell University Press. I have made a good-faith effort to procure permission for e-print to no avail.

Fig. 9. Here Penrose uses the cosmological notion of particle and observer, by dividing the universe into "those [particles] that he can see and those he cannot see." If, according to Rindler, observers were on or in particles-as-nebulae, the meaning of "see" is unambiguous—nebulae emit light. Anthropomorphized observers do not "see" from a perch on an elementary particle. However, only two sentences later, particles "that he can 'see' at any one time" is set in quotes, perhaps indicating Penrose's knowledge of the tension between the two particle concepts at work. The steady-state cosmology has no particle horizon (Fig. 10); all particles will eventually be observed. More than just inserting quotes, however, Penrose moves back to the field understanding of particles after an interjection during his presentation by John Archibald Wheeler. Part of Penrose's response was that "The simplest way to visualize the situation for, say, a finite system of particles is to consider the behaviour of the field as it proceeds along a null geodesic in space-time into the future or into the past. We can find a characteristic behaviour, and we single out one term which behaves as $1/r$."[40] However, in his figures, the particles/fields *did not* travel on

40. Ibid., 45, 45, and 47, resp.

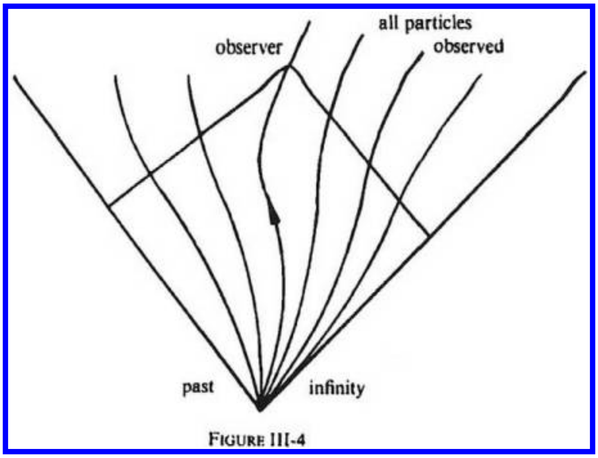

FIG 10. Penrose's (1963 [1967]) diagram of a cosmology without a particle horizon, "past infinity is null." *Source*: Penrose, "Cosmological Boundary Conditions" (ref. 33), 46. Copyright © 1967 by Cornell University. Used by permission of the publisher, Cornell University Press. I have made a good-faith effort to procure permission for e-print to no avail.

null (45°) lines. Penrose began this paper with particle fields organized by spin, but when discussing cosmology slipped into using "particle" and "observer" concepts from cosmology and astronomy that conflicted with this picture. "Particles" became stereoscopic, both nebulae and (sub-)nucleons. As he continued at the conference he pushed the formalist field theory concept, complete with tracing an isolated term in the field equation, together with the "particles" of his visualization.

This led Penrose to a discussion of the "initial value" problem of specifying how much information about the configuration of a system in the past you need to predict the system's evolution. In space-time, this information is arranged on a surface in the past of the system. Describing the merit of his discussion and diagrams he wrote: "One of the big advantages of being able to bring infinity to a finite place is that we can do the initial value problem for infinity in a perfectly rigorous way."[41] Trivially, this was about making some of physics well defined mathematically, which serves the cause of making further mathematical advances. But this was also about "placing" concepts, bringing

41. Ibid., 48.

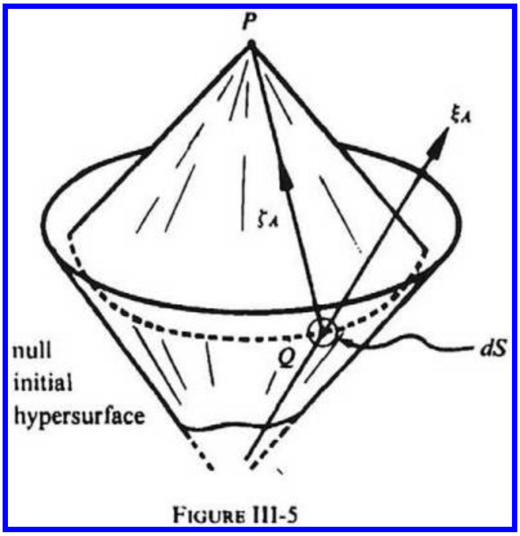

FIG 11. Penrose's (1963 [1967]) diagram for the initial value problem. *Source*: Penrose, "Cosmological Boundary Conditions" (ref. 33), 48. Copyright © 1967 by Cornell University. Used by permission of the publisher, Cornell University Press. I have made a good-faith effort to procure permission for e-print to no avail.

a concept like "infinity" to a "finite place" on the page, where it could be spatially related to other concepts (i.e., "field at a point"). The power of the method, which allowed GR to become more "rigorous," was "being able to bring infinity to a finite place." In the case of the initial value problem, Penrose's approach allowed him to "specify data on the bounding surface."[42] Here he introduced a diagram, Fig. 11, that included writing and labels; notably, some of these labels are variables in the accompanying equations. In earlier diagrams such as Fig. 6 there was labeling text and symbolic labels. These enter into the text of the paper, in clauses such as "specialize $g_{\mu\nu}$ so that $R = 12$ on $\mathscr{I}^+$," but there was no, say, $R = 12$ in Fig. 6.[43] In Fig. 11, in the context of a blurring between meanings of the dots and lines of the page between particle-fields and particle-nebulae, elements from the equations appear in the diagram. Geometric elements like the line *QP* were married to elements of formalism

42. Ibid.
43. Penrose, "Asymptotic Properties" (ref. 31), 67.

defined by integrals such as *dS* and the spinors ξ_A and ζ_A.[44] The diagrams were becoming denser. As earlier, there remain elements of the diagram that were more instructions to the reader than geometrical objects. Recall the instructions to "identify" parts of the earliest diagrams (Fig. 5); here the reader was supposed to take the bent arrow from *dS* to the intersection of ξ_A and ζ_A to be an aid to the audience that indicated the position of *dS* and should not be interpreted as a line that is part of the geometry of the space. This bricolage character of the diagrams—marrying geometric lines, text, mathematical symbols, instructional symbols, shading—continues to intensify, making the diagrams richer and denser. Though at the same time as they become denser, they transgress Goodman's distinction between description and depiction: the spinor ξ_A is part of the depiction and the description.[45]

In the text, formalism, and figures of Penrose's lectures at Ithaca, Penrose adapted his presentation to new contexts, blurring conceptual boundaries. Particles were defined in the language of particle physics as fields with spin, but were also written of as if they were home to observers in the language of cosmology. The lines on the page manifest this tension. Conformally invariant fields are massless—neutrinos and photons move at 45° on the diagram. Rindler's cosmological "fundamental particles" were galaxies with human observers, which moved bottom-to-top on bowed lines. Figure 10 illustrates the conceptual clash: particles qua formalism must move at straight 45° lines, but they do not.

Penrose won Cambridge University's Adams Prize with an essay that represents, recasts, and advances upon much of the work discussed above. It included reproductions of diagrams from his published lectures as well as hand-drawn diagrams. Much of the essay was later published, but not before it was mimeographed and circulated during his NATO postdoc in the United States. I would like to conclude this section with just one page of Penrose's essay, Fig. 12, with four diagrams depicting different ways of "manifesting" the

44. Here is Penrose's expression for the value of a general spinor-represented spin *s* field $\varphi_{AB\ldots L}$ in terms of an integral over the points, *Q*, that lie on the surface of intersection, *dS*, of the past-cone of *P* with the null initial hypersurface,

$$\phi_{AB\ldots L} = \frac{1}{2\pi}\int \frac{1}{r}\zeta_A\zeta_B\cdots\zeta_L\{(D\phi - 2s + 1)\rho\phi\}dS + \text{any source terms.}$$

45. Nelson Goodman, *Languages of Art: An Approach to a Theory of Symbols*, 2nd ed. (Indianapolis, IN: Hackett, 1976).

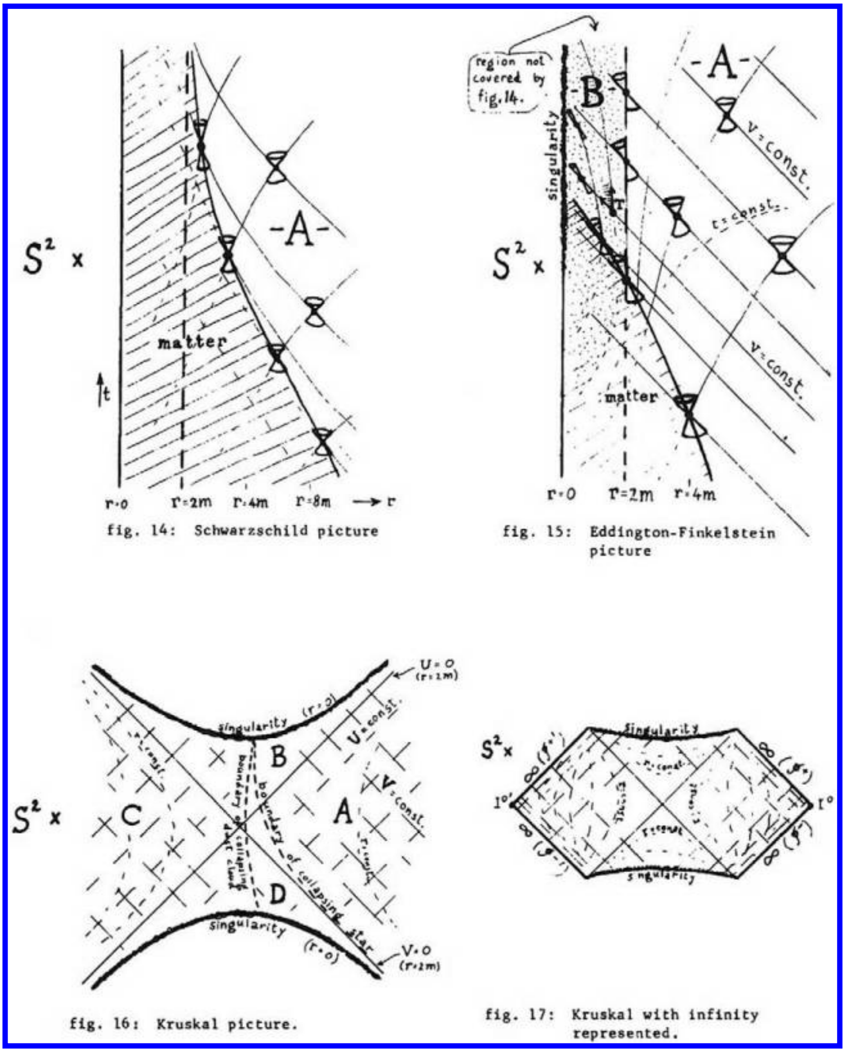

FIG 12. Four manifestations of the Schwarzschild solution. *Source*: Penrose, "Analysis of the Structure of Space-Time" (ref. 46), 687. Copyright Oxford University Press, reproduced with permission.

Schwarzschild metric of a collapsing star.[46] Following the diagrams left-to-right in two rows, we are given an historical evolution of how collapsing stars

46. Roger Penrose, "An Analysis of the Structure of Space-Time [Adam's Prize 1965–66]," chap. 28 in *Roger Penrose: Collected Works*, Oxford Science Publications (Oxford: Oxford University Press, 2011), 687.

were understood. Each new diagram offered a liminal space unseen between the regions of the previous diagram, except the last one, which manifested Penrose's contribution. Rather than revealing new spaces hiding behind black hole horizons, Penrose allowed the entire mathematical space—the entire star, an entire pair of infinite universes—to be placed on the page. In the first figure (Fig. 12 top left) Penrose illustrated the Schwarzschild solution to the Einstein Field Equations, which reads:[47]

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \frac{dr^2}{(1-2m/r)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{1}$$

The diagram showed two dimensions of four (t, r, θ, φ), and indicated that each point should be considered as the surface of a sphere (the θ, φ) by writing $S^2\times$ at the vertical t and horizontal r axis.[48] The shaded area represented the interior of the star. By inspecting Eq. 1 it is easy to see that there is a singularity where the term proportional to dr^2 becomes infinite at $r = 2m$, where m is the mass of the star. The area "–A–" represented regular space outside the star, and the hourglass figures are light-cones at space-time points. As one follows the circumference of the star from the bottom right of the diagram as it approaches the singularity at $r = 2m$, it appears that an observer would not cross this singularity. However, the "proper time" for an observer at the surface of the star to reach the singularity is "finite . . . thus, an observer who follows the star inwards must . . . either of necessity be destroyed . . . or else, find himself in a portion of the universe not covered by the coordinates of [Eq. 1]."[49] By introducing a new time parameter $v = t + r + 2m \log(r - 2m)$, a type of "analytic extension," the Schwarzschild metric became

$$ds^2 = (1 - 2m/r)dv^2 - 2\, dv\, dr - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{2}$$

This accorded to the second diagram (Fig. 12 top right) in Eddington-Finkelstein coordinates.[50] It was now possible to picture an observer on the surface of the star falling through the point at $r = 2\ m$ and reaching the singularity at the center of the coordinate system, $r = 0$. Area "–B–" was the

47. Jean Eisenstaedt, "Trajectoires et Impasses de la Solution de Schwarzschild," *Archive for History of Exact Sciences* 37, no. 4 (1987): 275–357.

48. Penrose, "Structure of Space-Time" (ref. 46), 686. Citations are to the collected work's pagination.

49. Ibid.

50. David Finkelstein, "Past-Future Asymmetry of the Gravitational Field of a Point Particle," *Physical Review* 110, no. 4 (1958): 965–67.

new area depicted in this diagram that is not included in the first diagram. Note that the light cones representing observers are elongated as in the first diagram but now gradually tilt into the singularity. The cones along the $r = 2$ m line with the rightmost future-directed (up) boundary on the line are at the point of no return from the collapsing star. As with the $r = 2\ m$ case, the proper time for an observer to reach the singularity at $r = 0$ is finite. Could another set of coordinates be found to carry an observer past this singularity? No, "our observer who successfully followed the star through r = 2 m must now be torn to pieces by the infinite tidal forces at the true singularity at r = 0."[51] Connecting this diagram to Oppenheimer and Snyder's 1939 discussion of a collapsing dust cloud, Penrose observed that the area inside the collapsing cloud—hashed and labeled "matter" in the top-right diagram—"turns out to be nothing other than (a portion of) the Friedmann universe," a cosmological model of a universe beginning from a singularity, expanding, and then contracting back to a singularity.[52] For Penrose this motivated "the essentially identical nature of the situation of a final singularity in gravitational collapse and of a final (or initial) singularity for a relativistic cosmology."[53]

The dual final-initial status presages Penrose's next investigation. He noted that the equations that govern a collapsing dust cloud (or star) are time-symmetric, just as the equations for a Friedmann cosmology.[54] "Thus we may expect to join the matter-filled region to an empty, spherically symmetrical exterior region which is also time symmetrical." This was accomplished by using Kruskal's U and V coordinates,[55] which were defined in terms of Schwarzschild coordinates (Eq. 1).[56] Then the Kruskal metric became

$$ds^2 = (32m^3/r)r^{-r/2m}\ dU\ dV - r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{3}$$

This gave the "Kruskal picture" (Fig. 12 bottom left) that shows an empty, infinite complement –C– to the region –A–. The diagram is of the UV-plane, and Penrose has labeled some lines to indicate how these relate to the original

51. Because "the curvature scalars constructed from the Weyl tensor . . . tend to infinity as r approaches zero." Penrose, "Structure of Space-Time" (ref. 46), 688.

52. J. R. Oppenheimer and H. Snyder, "On Continued Gravitational Contraction," *Physical Review* 56, no. 5 (1939): 455–59.

53. Penrose, "Structure of Space-Time" (ref. 46), 689.

54. Now called a Friedmann-Lemaître-Robertson-Walker cosmology.

55. M. D. Kruskal, "Maximal Extension of Schwarzschild Metric," *Physical Review* 119, no. 5 (1960): 1743–45.

56. $V/U = -e^{t/2m}$, $VU = e^{r/2m}(1 - r/2\ m)$. Penrose, "Structure of Space-Time" (ref. 46), 689.

Schwarzschild coordinates. The $U = 0$ and $V = 0$ diagonal lines are the "Schwarzschild radius" depicted as upright lines in the top diagrams. Neither U nor V can properly be called a time coordinate, but the diagram is (conventionally) oriented so that the future is up. The difference between a collapsing star and a collapsing cloud of dust is indicated labeled with dotted lines. As in the Eddington-Finkelstein case, the star begins in the distant past in region –A– and crossed the Schwarzschild radius $U = 0$ or $r = 2\ m$ to enter region –B– and ends at the future-singularity at $r = 0$. The dust cloud, however, begins as it ends—at a singularity—emerging from $r = 0$ into –D– passing briefly into –A– across $V = 0$ and then crossing $U = 0$ into –B– and ending at the future $r = 0$ singularity. At this stage in Penrose's paper, he had already introduced his conformal methods, and briefly remarks that it will be useful to have a version of the diagram with infinity represented as finite lines for a future discussion.[57] He referred to the last diagram (Fig. 12 lower right) with infinities in directions away from the singularities bounded by bold lines labeled ∞, and script "I"s, creating a Penrose diagram.[58]

Each diagram and its accompanying text formed a narrative structure, and the array of the four of them charts the understanding of ever-greater space-times. Observers, identified with hourglass light-cones, sit on the boundaries of stars. They were either torn to pieces or delivered to terra incognito, literally moving through interstices in the "Schwarzschild picture" to land in region –B– of the "Eddington-Finkelstein picture." Then connections between the astrophysics of black holes and cosmology and time symmetry arguments revealed a twin universe behind collapsing stars and dust clouds in the "Kruskal picture." These three diagrams were but pieces of the space-times they represent, which would continue infinitely above and below and to the right (and left for the Kruskal picture) if space allowed. Not so for the last diagram, "Kruskal with infinity represented"; the entire space-time with two infinite universes and a pair of singularities was present on the page, there to be the object of intuitions, to aid in understanding the empty and infinite.

The Context of Pedagogy

Thus far the focus has been on Penrose's diagrams in research settings: academic conferences and prestigious journals. But the diagrams were swiftly

57. Using coordinates $p = \tan^{-1} \sinh^{-1} V$, $q = \tan^{-1} \sinh^{-1} U$.

58. Penrose, "Structure of Space-Time" (ref. 46), 690.

moved into advanced GR pedagogy.[59] No more than two years from Penrose's first use of the diagrams in research, they were presented to advanced graduate students and young researchers at the 1963 Les Houches Summer School in theoretical physics. The annual school was founded by Cécile Morette in 1951 explicitly as part of the reconstruction of scientific institutions in postwar France.[60] More than simply moving immutably, the diagrams were changed within pedagogical contexts. They were mutable mobiles.[61] Even this boundary between research and pedagogical contexts—conference presentations versus "summer school" lectures—was breached. One of Penrose's lectures at Les Houches introduced a new, more general relativistic energy-momentum conservation law, illustrated with a diagram.[62] In fact, not only were Penrose's diagrams integrated into advanced pedagogy by Penrose himself, it is a testament to the influence of his work and to the rapid integration of research into pedagogy in GR that his diagrams appeared in others' lectures as well. Rainer K. Sachs—another rising young star of GR—introduced in his lecture a two-dimensional Minkowski space with metric $ds^2 = dt^2 - dx^2$. He cited Penrose's 1963 paper in *Physical Review Letters*.[63] Redefining things in terms of new variables u and v, such that $u = t+x$ and $v = t-x$, Sachs rewrote the metric $ds^2 = dudv$. He then applied the coordinate transformation $u = \tan u'$, $v = \tan v'$, "then we can for all of two dimensional Minkowski space . . . into a finite picture: [Fig. 13]."[64] This was the same space-time as the left side of Fig. 5 from Penrose's presentation in 1962 in Warsaw (a conference Sachs attended) and was in flat perspective, forcing Sachs to label two I°s, as opposed to the "consider as one point" of Fig. 6.[65] Sachs's diagram was sparer than Fig. 5, and did not represent the changing length scales across the diagram; it was

59. See Buhm Soon Park, "In the 'Context of Pedagogy': Teaching Strategy and Theory Change in Quantum Chemistry," in Kaiser, *Pedagogy* (ref. 18) for more on the "context of pedagogy."

60. Interview with Drs. Bryce DeWitt and Cécile DeWitt-Morette, by Kenneth W. Ford, at University of Texas at Austin, 28 Feb 1995, www.aip.org/history/ohilist/23199.html (accessed 27 Sep 2013). See also John Krige, *American Hegemony and the Postwar Reconstruction of Science in Europe*, Transformations (Cambridge, MA: MIT Press, 2006).

61. Kaiser, *Drawing Theories Apart* (ref. 21), 7.

62. Penrose, *Roger Penrose: Collected Works* (ref. 46), 446.

63. R. K. Sachs, "Gravitational Radiation," in *Relativity, Groups, and Topology: Lectures Delivered at Les Houches during the 1963 Session of the Summer School of Theoretical Physics*, ed. C. DeWitt and B. DeWitt (New York: Gordon and Breach, 1964), 536.

64. Ibid., 539–40.

65. Infeld, *Conférence Internationale* (ref. 28), ix.

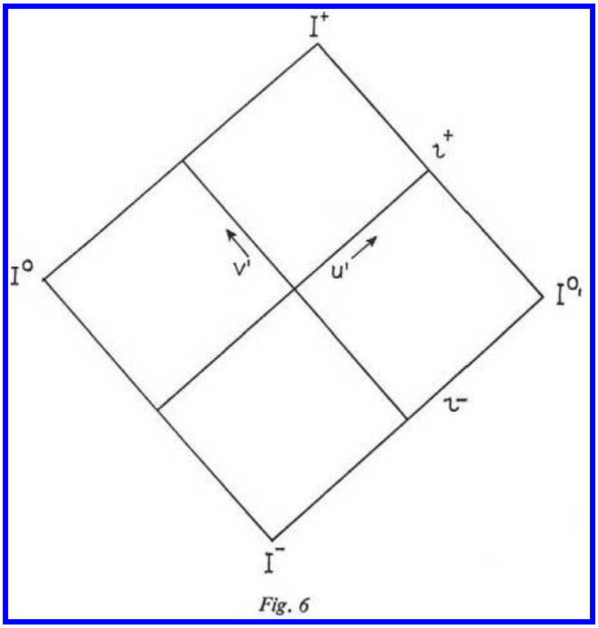

FIG 13. Sachs's conformal Minkowski space (1964). *Source*: Sachs, "Gravitational Radiation," in DeWitt and DeWitt, *Relativity, Groups, and Topology* (ref. 63), 536. I have made a good-faith effort to procure permissions to print this figure but to no avail.

embellished with the labels of the parts of the space-time (I^+, I^-, I°, $I^{\circ\prime}$, i^+, i^-), and with the coordinates in the equations for the conformal transformation, similar to the ξ and ζ spinors in Fig. 11. The focus shifted from depicting the asymptotic character of the diagram to the relation between the diagram and the formalism—i.e., on how to make the diagrams.

Penrose's diagrams at Les Houches were characterized by the increasing presence of elements of formalism in the diagrams. He began his lectures with a more abstract diagram than he had previously used—abstract in the sense that he did not introduce a specific metric or line element and manifold, but rather a general "physical" manifold $\tilde{\mathscr{M}}$ imbued with an unspecified metric $\tilde{d}s$.[66] "The idea is to construct another 'unphysical' manifold M with a boundary I and metric ds, such that $\tilde{\mathscr{M}}$ is conformal to the interior of $\mathscr{M}$ with $ds = \Omega \tilde{d}s$,

66. Roger Penrose, "Conformal Treatment of Infinity," in DeWitt and DeWitt, *Relativity, Groups, and Topology* (ref. 63), 565.

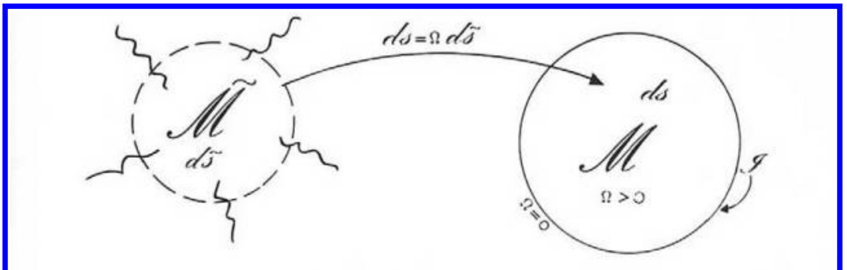

Fig. 1. The infinite physical space-time $\tilde{\mathscr{M}}$ is mapped into an unphysical "finite" conformally equivalent manifold $\mathscr{M}$, with boundary $\mathscr{I}$ corresponding to the "infinity" of $\tilde{\mathscr{M}}$.

FIG 14. Penrose's opening diagram at Les Houches. The caption reads: "The infinite physical space-time M̃ is mapped into an unphysical 'finite' conformally equivalent manifold M, with boundary I corresponding to the 'infinity' of M̃." *Source*: Penrose, "Conformal Treatment of Infinity" (ref. 66), 565. I have made a good-faith effort to procure permissions to print this figure but to no avail.

and so that the 'infinity' of $\tilde{\mathscr{M}}$ is represented by the 'finite' hypersurface $\mathscr{I}$" (see Fig. 14).

The quotes may be interpreted as indicating suspension of disbelief involved in being able to point to a line and call it "infinity," though this was a curious inversion of the disbelief required to imagine a "physical" infinite expanse (Fig. 14). The quotes reveal a tension at the heart of Penrose's diagrammatic papers. Figure 14 was relatively spare; however, the next diagram, Fig. 15, was richly shaded and shown in perspective. Even as he announced in the text that these objects are "unphysical," his diagrams exhibited a realism. On the one hand the mathematical context of conformal and coordinate transformations must be remembered, on the other the diagrams brought infinite objects almost within reach. Every element in Penrose's formal discussion thus far had been integrated into the diagram. The five rippled lines emanating from the initial (left) pair of mathematical objects (manifold and metric) indicate that they are supposed to be infinite in expanse (Fig. 14). The central arrow shows the action done by multiplying the metric $d\tilde{s}$ by the conformal factor Ω. Another arrow serves as a marker to indicate the "'finite' hypersurface $\mathscr{I}$." What makes $\mathscr{I}$ infinity? As indicated on the diagram, "the condition that $\Omega = 0$." This intertwining of diagram (Fig. 14) and formalism demonstrates their inseparability. With this apparatus, properties of $\tilde{\mathscr{M}}$ and fields on $\tilde{\mathscr{M}}$ as they extend asymptotically to infinity "can now be investigated by studying $\mathscr{I}$, and the *local* behaviour of the fields at $\mathscr{I}$—provided that all the relevant concepts can be put into

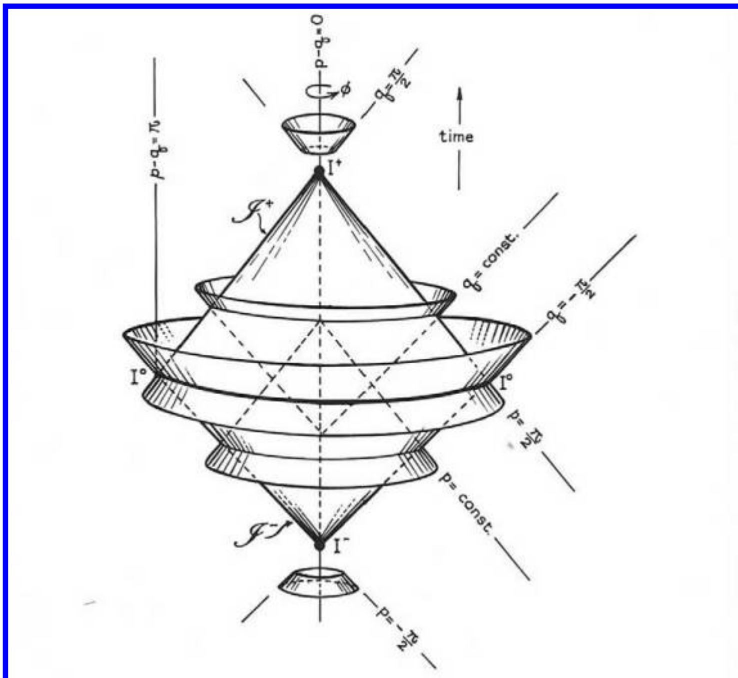

Fig. 2. Coordinates for $\mathscr{M}$ in Minkowski case. The θ coordinate has been suppressed ($\theta = \pi/2$).

FIG 15. Penrose's diagram of the relationship between *u*, *v* advanced and retarded time coordinates for Minkowski space and the conformally transformed *p*, *q* coordinates. *Source*: Penrose, "Conformal Treatment of Infinity" (ref. 66), 568. I have made a good-faith effort to procure permissions to print this figure but to no avail.

a conformally invariant form."[67] This conceptual work is done with formal tensorial and spinorial manipulation of zero rest-mass fields that appear in later diagrams. Previously Penrose had taken two formal steps in one diagram, depicting a conformally transformed space-time that also underwent a coordinate transformation. At Les Houches, he provided one diagram for each formal step, Fig. 14 showing the conformal transformation Ω and Fig. 15 showing the relation between coordinate systems.

Penrose moved from this abstract representation to again draw a conformally transformed Minkowski space-time, maintaining his skilled use of perspective in Fig. 6 but now tightly integrating the diagram with the formalism. In this pedagogical context he also multiplied the representations to include examples

67. Ibid., 565.

of space-times that would be familiar to the students at Les Houches (Figs. 15, 16, and 17). Penrose began by writing the Minkowski metric in terms of null coordinates (as in the discussion of Sachs's diagram, Fig. 13),

$$d\tilde{s} = dudv - \frac{1}{4}(u-v)^2\{d\theta^2 + \sin^2\theta\ d\phi^2\}, \quad u \geq v, \tag{4}$$

and provided a "convenient conformal factor . . . $\Omega = (1 + u^2)^{-1/2}(1 + v^2)^{-1/2}$." He then introduced new coordinates p and q, related to u and v as "$u = \tan p$, $v = \tan q$," then "the unphysical metric takes the form

$$ds^2 = \Omega^2 d\tilde{s}^2 = dpdq - \frac{1}{4}\sin^2(p-q)\{d\theta^2 + \sin^2\theta d\phi^2\}, \tag{5}$$

where p and q vary from $-\pi/2$ to $\pi/2$ with $p \geq q$." Penrose then defined the various pieces of the space-time—$\mathscr{I}^{\pm}$, $I^{\pm}$, I°—in terms of ranges of the coordinates p, q and refers to Fig. 15. This figure is very different from Penrose's 1963 diagram in *Physical Review Letters* (Fig. 6), though it maintains perspective, allowing it to represent three of the four coordinates (p, q, φ not θ in Eq. 5). The coordinates in Fig. 15 are indicated by the lines "$p =$ const." and "$q =$ const." and the counterclockwise arrow at the top of the diagram labeled φ. The time coordinate is obscured inside p and q, but Penrose indicated the forward time direction with an arrow. Recall that where the boundary of the conformal space is zero, "the [physical] metric . . . is stretched by an infinite factor" (it may be helpful to imagine taking the limit of $1/x$ as x goes to zero). Then in Fig. 15, I° is an "infinite" point when $p - q = \pi$ because $\sin^2(p - q)$ in the metric, Eq. 5, is zero. When $p = \pi/2$, the original u coordinate given by $u = \tan p$ becomes infinite; Ω is proportional to $1/u$, so at any value of q with $p = \pi/2$, $\Omega = 0$, which is the condition for $\mathscr{I}$ from the discussion of Fig. 14. Hence "let $\mathscr{I}^+$ be given by $p = \pi/2$, $-\pi/2 < q < \pi/2$."[68] The formalism in the text gave little insight to the geometry of objects such as $\mathscr{I}^+$, but the drawing obscured some facets of the mathematics (e.g., I° is a point); together they gave an understanding of conformally transformed space-time that combined physical intuition about surfaces like cones with formal mathematical understanding.

But Fig. 15 was not alone; at Les Houches Penrose included two complementary ways of picturing the space-time with metric Eq. 5, one (Fig. 16) depicting the new conformal space-time as a part of a more familiar space-

68. Ibid., 565, 568, 565, and 567, resp.

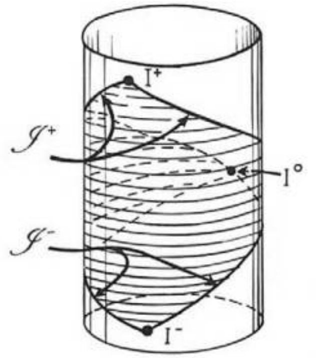

Fig. 3. The cylinder represents the Einstein universe. The two spacial coordinates θ, ϕ have been suppressed; only one space and one time dimension are depicted. The shaded region corresponds to the manifold $\mathscr{M}$.

FIG 16. Conformal space-time wrapped on the Einstein universe. *Source*: Penrose, "Conformal Treatment of Infinity" (ref. 66), 569. I have made a good-faith effort to procure permissions to print this figure but to no avail.

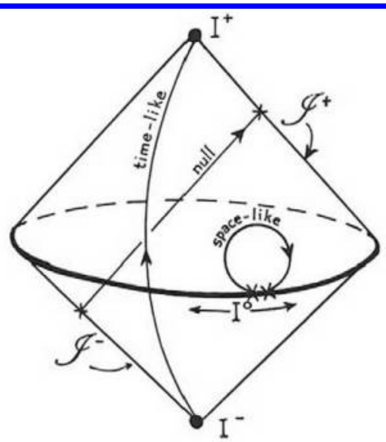

Fig. 4. The images of straight lines in $\tilde{\mathscr{M}}$. (One space dimension is suppressed).

FIG 17. The images of straight lines in the "physical" metric M̃. *Source*: Penrose, "Conformal Treatment of Infinity" (ref. 66), 569. I have made a good-faith effort to procure permissions to print this figure but to no avail.

time, the other (Fig. 17) abandoning specific reference to coordinate systems to depict how straight lines change under the conformal and coordinate transformations. Turning briefly to Fig. 16, Penrose wrote that the original metric "is that of an Einstein static universe—a cylinder ($S^3 \times E^1$) which represents a three-dimensional spherical space which is constant in time." This diagram was in three-dimensional perspective, but was a two-dimensional depiction of the space-time, on the surface of the cylinder. With this way of seeing, Penrose explained that I° has been "refocused to a point." Fig. 17 was an opposite move to the increasing complexity of these diagrams. It was still in three-dimensional perspective but lost the labels relating the manifold to its coordinate system. Instead, text labeled types of lines in the original, infinite space-time. It is in this sparer representation that the "meaning of I^{-}, $\mathscr{I}^{-}$, I°, $\mathscr{I}^{+}$, I^{+} [was] seen by considering the behaviour of curves in $\mathscr{M}$ corresponding to straight lines in $\tilde{\mathscr{M}}$." The path of an unaccelerating observer in Minkowski space is an upright (time-like) straight line—here transformed into an upright curve beginning at I^{-} and ending at I^{+}. A plane of simultaneity in Minkowski space is horizontal (space-like), and becomes a circle beginning and ending at I°. Paths of light travel in straight, 45°lines, but in the conformal space-time begin on $\mathscr{I}^{-}$ and end on $\mathscr{I}^{+}$. "Thus, at I^{-} represents [*sic*] past infinity; I° represents spatial infinity; I^{+} represents future infinity; $\mathscr{I}^{-}$ represents past *null* infinity; $\mathscr{I}^{+}$ represents future *null* finity [*sic*]."[69]

This was in the same pedagogical register as the diagram of the conformal space-time wrapped on the Einstein universe—explicating in detail the relationship between the new mathematical object and objects students could be expected to be familiar with (at least from the other lectures at the summer school). Penrose then explained that this demarcates what regions will be important to different fields. He wrote "that zero rest-mass fields are to be significant at $\mathscr{I}^{-}$ and $\mathscr{I}^{+}$, but that fields of finite rest-mass are important at I^{-} and I^{+} and not at $\mathscr{I}^{-}$ and $\mathscr{I}^{+}$."[70] Here again the conceptual line between the traditional cosmology of the Einstein universe and the particle physics of "zero rest-mass fields" was blurred.

Penrose's lectures at Les Houches blurred the boundary between research work and pedagogical work in the speed with which cutting-edge work was taught to students. He further blurred this line by giving a new, more general formulation of the energy-momentum conservation law in GR; this pedagogical venue saw results not yet aired in standard research settings. This new formulation came in Penrose's second lecture, and was accompanied by a diagram that exhibits not only the structure of a space-time, but also represents

69. Ibid., 567.
70. Ibid.

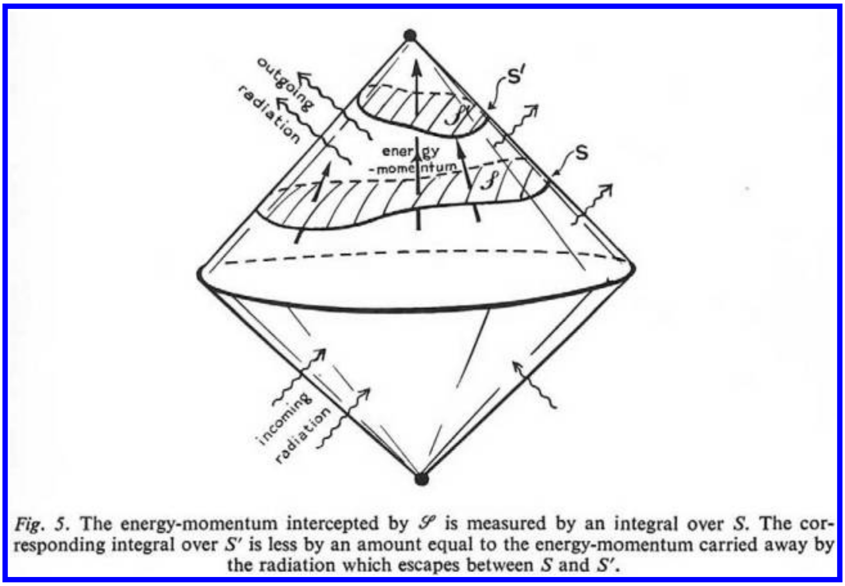

FIG 18. Penrose's novel picture of energy-momentum conservation. *Source*: Penrose, "Conformal Treatment of Infinity" (ref. 66), 573. I have made a good-faith effort to procure permissions to print this figure but to no avail.

the *fields* defined over the space-time. I am passing over many details, but Penrose considered fields in a conformal space-time in a similar way to his paper given at Cornell that spring.[71] "The energy-momentum intercepted by $\mathscr{S}$ is measured by an integral over *S*. The corresponding integral over *S*′ is less by an amount equal to the energy-momentum carried away by the radiation which escapes between *S* and *S*′."[72]

Here $\mathscr{S}$ ($\mathscr{S}'$) was a hypersurface in M that intersected the future null cone along some surface *S* (*S*′).[73] The novelty of Penrose's work was that the two surfaces on which he measured the fields do not need to be parallel.[74] Instead of directly placing parts of formalism in the diagram as in Fig. 14, this diagram, Fig. 18, used different styles of arrows and textual labels to give meaning to the concept of incoming and outgoing radiation. These simple arrows represented

71. Penrose, "Cosmological Boundary Conditions" (ref. 33).

72. Penrose, "Conformal Treatment of Infinity" (ref. 66), 573.

73. In the diagram *S* and *S*′ are lines, but in the full space-time they have the topology of spheres.

74. Compare Sachs, "Gravitional Radiation" (ref. 63), 525; Penrose, *Roger Penrose: Collected Works* (ref. 46), 447.

expressions of daunting complexity, expressed in Penrose's only recently developed spinor formalism.[75]

The upward arrows provided a succinct and direct sense of what Penrose's intricate formalism meant; they allowed a physicist to have an intuition about a tangle of spinors.[76] Penrose wrote that the difference in total energy momentum at $\mathscr{S}$ and $\mathscr{S}'$ "can be expressed as an integral over the portion of $\mathscr{I}^+$ lying between S and S' of $N\bar{N}W_\mu$ (representing the gravitational energy flux)." The diagram explained this with the three undulating arrows leaving the null cone between the surfaces, without recourse to spinors.[77] Penrose expressed a novel extension of the complex concept of energy-momentum conservation in GR with intricate formal expressions paired with simple diagrammatic elements that gave an intuitive understanding of this result.

FURTHER RECEPTION OF THE DIAGRAMS

The diagrams moved out into the community of physicists similarly to how they were introduced: within a close admixture of research and pedagogy. Here I will discuss chronologically some notable examples of their use, not a complete analysis. This will show the diagrams' circulation among physicists on both sides of the Atlantic and also chart their movement from conference presentations to their integration into highly regarded textbooks and monographs. Penrose next presented his diagrams at a conference that was not aimed at advanced students, but was a pedagogical gathering for researchers at many

75. Roger Penrose, "A Spinor Approach to General Relativity," *Annals of Physics* 10, no. 2 (1960): 171–201. The total of the three strong upward-pointing arrows labeled "energy-momentum" that pierce S is equal to "the following integral over *S*:

$$P_\mu = \frac{1}{4\pi}\int(\sigma N - \psi_2)W_\mu dS$$

where *dS* is an element of surface area of *S*." In terms of a spinor field characterizing the gravitational field φ_{ABCD}, the Ricci tensor $R_{\mu\nu}$, the Pauli matrices σ^μ, and various spinors corresponding to the null directions in the space-time,

$$\begin{aligned} \text{``}\psi_2 &= \phi_{ABCD}\eta^A\eta^B\xi^C\xi^D \\ N &= -\tfrac{1}{2}R_{\mu\nu}\sigma^\mu_{A\dot{B}}\sigma^\nu_{C\dot{D}}\eta^A\bar{\xi}^{\dot{B}}\eta^C\bar{\xi}^{\dot{D}} \\ \sigma &= \xi^A\bar{\eta}^{\dot{B}}\xi^C\nabla_{A\dot{B}}\xi_C\text{,''} \end{aligned}$$

and W_μ is a weighting factor. Penrose, "Conformal Treatment of Infinity" (ref. 73), 574.

76. Perhaps drawing from the convention of drawing arrows to represent forces or momenta in Newtonian mechanics.

77. Penrose, "Conformal Treatment of Infinity" (ref. 66), 574.

levels. The conference was organized by C. DeWitt and Wheeler and held at the Battelle–Seattle Center at the University of Washington; it was lightheartedly called *Battelle Rencontres*. The context in Seattle was pedagogy given to established workers in the fields of mathematics and physics. The explicit aim of the conference was laid out in the organizers' introduction to the published proceedings "Battelle and Babel": it was devoted to "widening the lines of communication between workers in mathematics and workers in physics." And the "purpose was frankly pedagogical."[78] Penrose's paper, "Structure of Space-time," laid out introductory material such as "The Nature of General Relativity" and spent time explaining to mathematicians the notation physicists used. His discussions of his conformal methods, horizons, and gravitational collapse made extensive use of the diagrams. He continued to use them in research contexts in journal publications and locally circulated mimeographs during a series of visiting appointments at American universities.[79] The diagrams were also used in his work with Stephen Hawking.[80] Penrose's *Collected Works* contains an extensive bibliography.[81]

Brandon Carter, another Cambridge-trained physicist, made extensive use of the diagrams and Penrose's conformal techniques in both research and—again at Les Houches—in the pedagogical context of summer schools. Previously, Penrose had applied his method to Minkowski space-time, different cosmological models, and the Schwarzschild space-time. The latter represented how space-time is bent by spherically symmetric distributions of matter, like idealized stars. Manipulations of the Schwarzschild picture were also the primary way Penrose approached black holes. In two influential papers in 1966 Carter applied conformal methods and diagrams to space-time more complicated than Minkowski space-time.[82] These space-times—Kerr and Reissner-Nordström—

78. Cécile DeWitt-Morette and John Archibald Wheeler, "Battelle and Babel," in *Battelle Rencontres: Lectures in Mathematics and Physics*, ed. Cécile DeWitt-Morette and John Archibald Wheeler (New York: W. A. Benjamin, 1968), ix–xii.

79. See for example Penrose, "Structure of Space-Time" (ref. 46); E. T. Newman and R. Penrose, "New Conservation Laws for Zero Rest-Mass Fields in Asymptotically Flat Space-Time," *Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences* 305, no. 1481 (1968): 175–204.

80. Hawking and Penrose, "Singularities of Gravitational Collapse" (ref. 2).

81. Penrose, *Roger Penrose: Collected Works* (ref. 46).

82. Brandon Carter, "The Complete Analytic Extension of the Reissner-Nordström Metric in the Special Case $e2 = m2$," *Physics Letters* 21, no. 4 (1966): 423–24; Brandon Carter, "Complete Analytic Extension of the Symmetry Axis of Kerr's Solution of Einstein's Equations," *Physical Review* 141, no. 4 (1966): 1242–47.

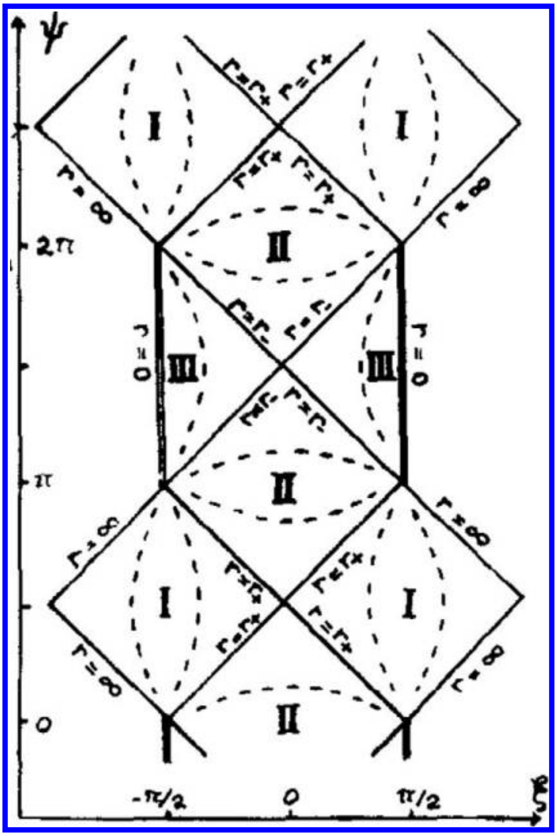

FIG 19. Carter's diagram of the Reissner-Nordström space-time (1966). *Source*: Carter, "Reissner-Nordström Metric" (ref. 82), 424. Copyright © 1966 Elsevier, reproduced with permission.

were taken to be more general versions of the Schwarzschild space-time. For example, instead of the Schwarzschild situation of an idealized star that is perfectly round and stationary, the Kerr picture described a star that could rotate. The difference Penrose's conformal methods made can be illustrated by a comparison of Carter's diagram (Fig. 19), which depicted a portion of a braided chain of infinite universes, with a diagram from Graves and Brill from 1960, before Penrose's work (Fig. 20).[83] Graves and Brill's diagram was of-a-kind with Kruskal,

83. John C. Graves and Dieter R. Brill, "Oscillatory Character of Reissner-Nordström Metric for an Ideal Charged Wormhole," *Physical Review* 120, no. 4 (1960): 1507–13.

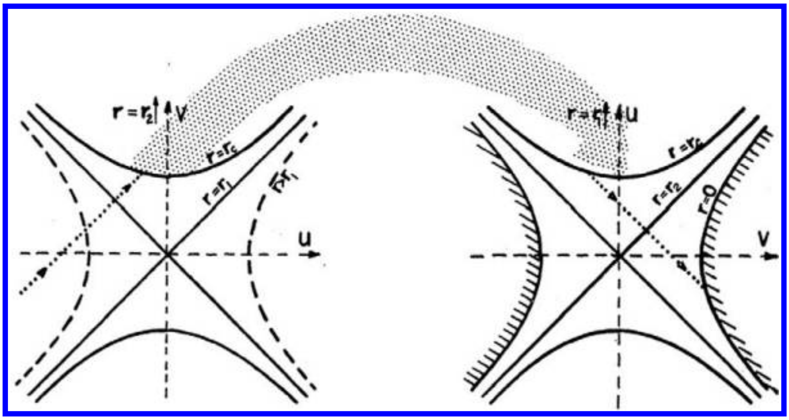

FIG 20. Graves and Brill's diagram of the Reissner-Nordström space-time (1960). *Source*: Graves and Brill, "Reissner-Nordström Metric" (ref. 83), 1511. Copyright © 1960 American Physical Society, reproduced with permission; see http://prola.aps.org/abstract/PR/v120/i4/p1507_1.

with infinite expanses left to the imagination at each corner of the diagram. This made it impossible to connect their two diagrams smoothly where $r = r_c$ at the top of each figure. It was left to a shaded arrow and the imagination of the reader to connect the two "patches" of the space-time. For Carter, Penrose's conformal methods allowed him to draw each patch as closed and connectable.

In 1972 Cécile and Bryce DeWitt invited Carter to speak at the August summer school at Les Houches on *Black Holes*, or in the more evocative French, *Les Astres Occlus*. His paper was entitled "Replication of Black Hole Equilibrium States."[84] Carter divided his lecture into two parts, first treating the specific case of the Kerr space-time and then developing a more general theory. In the sixty-seven pages of the printed proceedings of this first part he used more than twenty conformal diagrams. Though he was not the only one at the school to use the diagrams (Stephen Hawking used many as well), Carter's presentation, combined with his 1966 papers, were influential enough that what I have called "Penrose diagrams" have also been called "Carter-Penrose diagrams."[85]

84. Brandon Carter, "Republication of Black Hole Equilibrium States," *General Relativity and Gravitation* 41, no. 12 (2009): 2873–938; Brandon Carter, "Republication of Black Hole Equilibrium States—Part II: General Theory of Stationary Black Hole States," *General Relativity and Gravitation* 42, no. 3 (2010): 653–744.

85. Precise distinctions—or lack thereof—between "Penrose diagrams," "Carter-Penrose diagrams," and "conformal" diagrams are beyond the scope of this paper.

In a familiar pattern in the history of modern physics, Penrose diagrams and conformal methods of developing an understanding of general relativity were incorporated into textbooks and monographs. The most important of these books were both published in 1973: John Archibald Wheeler, Charles W. Misner, and Kip S. Thorne's *Gravitation*, a textbook meant to give graduate students an up-to-date and complete command of the field; and Stephen W. Hawking and George F. R. Ellis's *The Large-Scale Structure of Space-Time* (Cambridge monographs on mathematical physics).[86]

CONCLUSION

Penrose diagrams, like other scientific images, do not sit quietly within the realm of the visual—rather they trespass conceptual and disciplinary boundaries. In the first instance, Penrose diagrams came to be increasingly integrated with text in the figures. They were dotted with labels, equations, and instructions to the viewer. It is fitting that diagrams that unify space and time should transgress G. E. Lessing's classic division between the temporal seriality of the word and the spatial display of the image.[87] Pushing further on the interrelation of formalism and image, the analysis above supports the contention that they are in a strong sense inseparable. On the one hand, understanding or creating a diagram requires following links between the image (and textual elements in the image) and the accompanying text and formalism, and, though my survey of the literature is not complete, in the subfield of GR dealing with topology and causal connection, the diagrams are ubiquitous. With Bender and Marrinan, we can speak of "visual correlation as a form of knowledge."[88] And while we can agree with Nelson Goodman about the conventional and constructed nature of diagrams, this emphasis on correlation and interconnection erodes Goodman's distinction between description and depiction.[89] The

86. Misner et al., *Gravitation* (ref. 4); Hawking and Ellis, *Large-Scale Structure* (ref. 4).

87. W. T. J Mitchell, "Word and Image," in *Critical Terms for Art History*, ed. Robert Nelson and Richard Shiff (Chicago: University of Chicago Press, 1996), 51–61; Gotthold Ephraim Lessing, *Laocoon: An Essay upon the Limits of Painting and Poetry*, trans. Ellen Frothingham (Boston: Roberts Bros., [1766] 1887), at www.archive.org/details/laocoonessayupon00lessrich.

88. John B. Bender and Michael Marrinan, *The Culture of Diagram* (Stanford, CA: Stanford University Press, 2010), 13. Though I believe they go too far in stretching their analysis to include antivisual work in quantum mechanics; their "culture of diagram" should not be reduced to a culture of cross-reference, devoid of images.

89. David Kaiser, "Stick-Figure Realism" (ref. 17), 77.

formalism, though composed of discrete elements, does not describe the image; the image, though dense and in a sense indivisible, does not depict the formalism.[90] Rather the two elements combine to create understanding. Though it is not an economical use of space in this historical analysis, perhaps "diagram" should refer to the full page of a diagram embedded in the text of an article, say. This interconnectivity also denies the reducibility of image to text or symbol system—physicists already have a precise formal system related to the image: the mathematics on the page. The image and formalism come together as a diagram because each provides something the other does not.

But there is something else about Penrose diagrams that rejects a reduction to symbols; as objects of pedagogy, Penrose diagrams must be seen as prompts for others to re-create them. It does not make sense to separate material culture from visual culture. Images simply are material, whether captured in photographic emulsion or drawn in students' notebooks.[91] With Hans Belting we can reject Baudrillard's division of image and reality.[92] The understanding Penrose diagrams gave was in part haptic. Connections between aspects of infinity were *literally* drawn, and the physics student literally placed a universe on a page. This puts them within the multifaceted analytic purview of the "iconic turn" of visual studies, which broadened the field from attention only to sight.[93] A further elucidation of the place of Penrose diagrams within visual culture studies is a future project.

This paper has charted the evolution and circulation of Penrose diagrams from their introduction in 1962 to their inclusion in textbooks and monographs in 1973, focusing on the period 1962–66. It has demonstrated both the conceptual shifts engendered by the diagrams and the shifts and contradictions embodied within them.

Beginning in 1962 in Warsaw, the diagrams were used to reconfigure the basic relativistic concepts of space, time, and distance. Their use matured in a research publication, *Physical Review Letters*, where the conceptual structure changed, focusing on the nature of infinity. Moving from Warsaw to *PRL* the

90. Bender and Marrinan, *Culture of Diagram* (ref. 88), 6–7.

91. Peter Galison, *Image and Logic: A Material Culture of Microphysics* (Chicago: University of Chicago Press, 1997); James Elkins, "Logic and Images in Art History," *Perspectives on Science* 7, no. 2 (1999): 151–80; Peter Galison, "Refections on Image and Logic: A Material Culture of Microphysics," *Perspectives on Science* 7, no. 2 (1999): 255–84.

92. Though perhaps we do not need the help. Quoted in Keith Moxey, "Visual Studies and the Iconic Turn," *Journal of Visual Culture* 7, no. 2 (2008): 138.

93. Ibid.

style of depiction of the diagrams changed from more mathematical to realistic images of geometrical objects. At Cornell the context of the diagrams changed again—this time being adapted to discussions of relativistic cosmological models. Here a tension between the constraints of the formalism and the lines on the page was manifest. Penrose's formal language spoke of the massless subatomic "particles" of particle physicists traveling at the speed of light, while his diagrams depicted the massive "particles"-as-nebulae of cosmologists and astronomers. They were maps of the universe. And they were boundary objects trying to speak the language of two communities of scientists at once. In his Adams Prize essay, Penrose depicted an array of space-times showing how his progressive mathematical representations revealed new spaces about which to think. Each diagram presented a narrative about observers falling into a black hole; the array of diagrams presented a narrative about the evolution of the field of GR. Increasingly vast spaces were opened to analysis. Penrose's work in the context of research was characterized by his use of diagrams to place concepts that he sought to refigure. This was carried over into his pedagogy. Hard upon the introduction of the diagrams in 1962, Penrose adopted his diagrams to a pedagogical context at the 1963 Les Houches Summer School. Here the emphasis was on the interrelations of the formalism and the picture that would allow students to reproduce the diagrams on their own. The quick tempo of the introduction of this material into pedagogy reveals the closeness of research and pedagogy in the GR community in the 1960s. This closeness may be a clue as to why GR flourished in these years. Penrose diagrams were used to create new sorts of understanding and intuitions about General Relativity. This paper examined how concepts were reconfigured by the diagrams and how the diagrams were reconfigured themselves. The methodology has been close readings of published texts and diagrams. As the diagrams changed through time, they shifted the meanings of the objects they depicted, allowing a close reading of the changing work done by the diagrams to perform an historical investigation of ontological change in theoretical physics. While tracing these material traces of ontology, following Penrose diagrams revealed an aspect of the structure of the community of physicists who studied relativity: the close interrelation of advanced pedagogy and research. For physicists and as a contribution to the historiography of theoretical physics, then, this paper has tried to explicate the advantages of bringing infinity to a finite place.

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